

SAARC Training Workshop 'Power System Studies for Synchronization of Multiple Systems'

Symmetrical Components Theory

Day-2-D

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Symmetrical Components Theory

Unbalanced System

- Figure shows an unbalanced system where phase magnitudes and angles are unequal

$$|V_A| \neq |V_B| \neq |V_C|$$

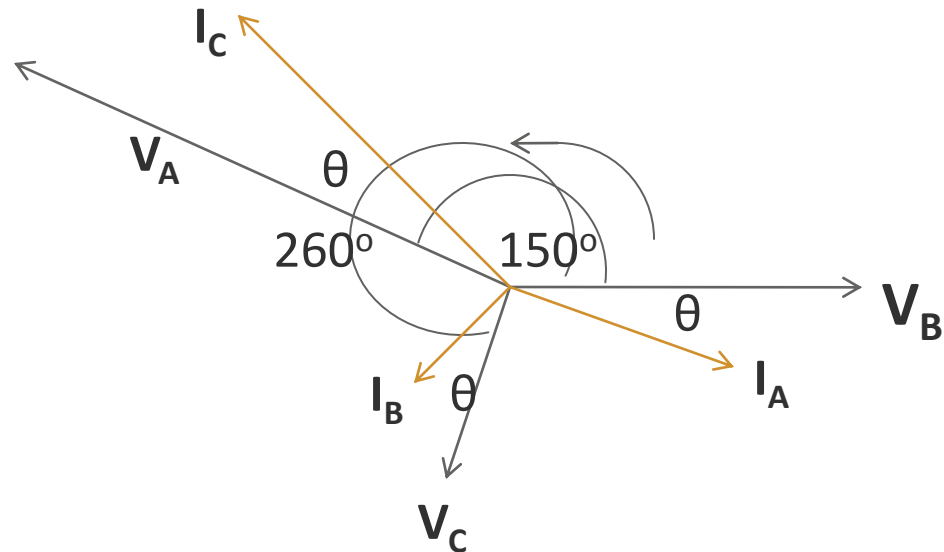
$$|I_A| \neq |I_B| \neq |I_C|$$

$$|Z_A| \neq |Z_B| \neq |Z_C|$$

- The formulae of balanced 3-phase system are not valid such as

$$V_{\text{Phase}} = \frac{V_{\text{L-L}}}{\sqrt{3}}$$

$$MVA = \sqrt{3} V_{\text{L-L}} \times I$$



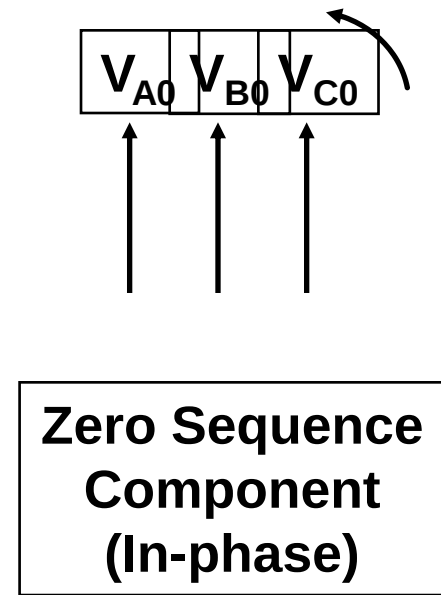
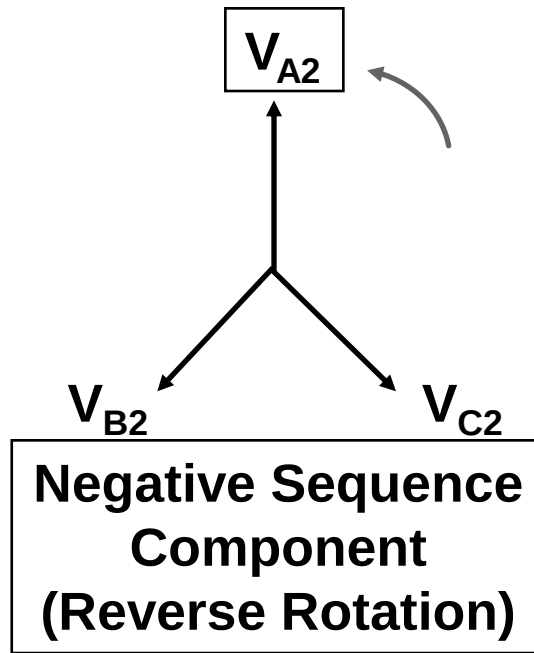
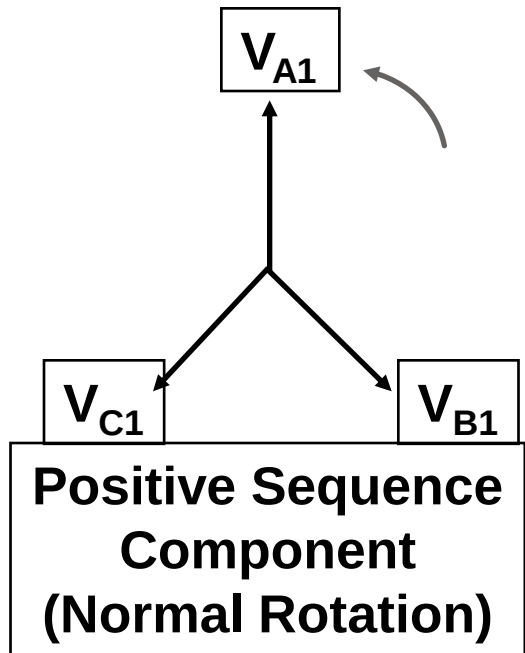
Symmetrical Components Theory

Background

- C.L. Fortesque, an American mathematician, consultant to Westinghouse, wrote a paper in 1918 and developed a mathematical model transforming unbalanced 3-phase vectors into a system of balanced symmetrical components
- An unbalanced 3-phase system was resolved into three systems of balanced symmetrical components
 - Positive sequence
 - Negative sequence
 - Zero sequence
- Each system can be treated separately just as balanced 3-phase problems were solved in the past by reducing the constants and voltages to per-phase values and solving on 1-phase basis
- It applies equally to voltages and currents
- All components rotate counter clock wise like usual Phase quantities

Symmetrical Components Theory

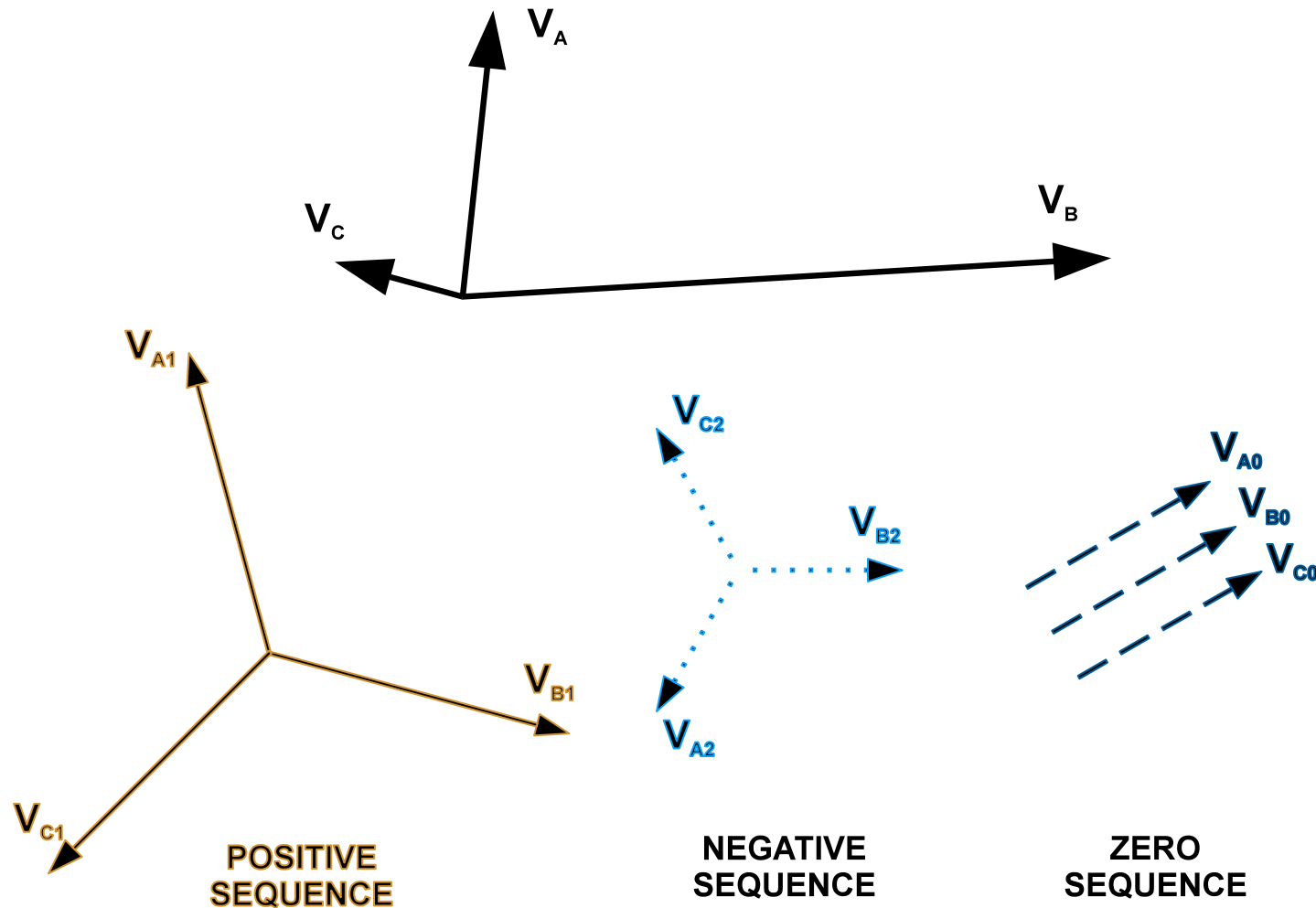
Positive, Negative and Zero Sequences



$$\begin{aligned} V_A &= V_{A1} + V_{A2} + V_{A0} \\ V_B &= V_{B1} + V_{B2} + V_{B0} \\ V_C &= V_{C1} + V_{C2} + V_{C0} \end{aligned}$$

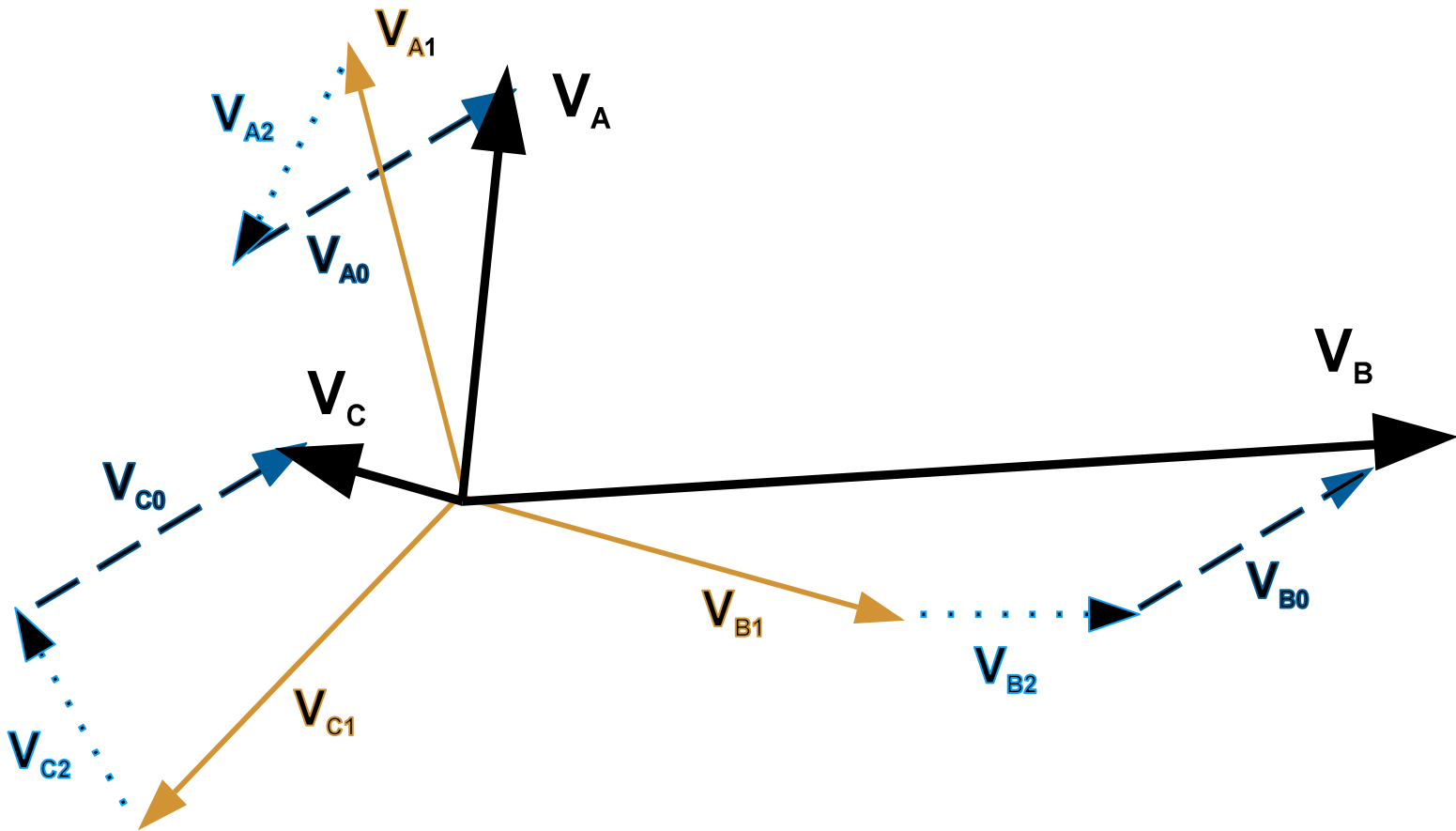
Symmetrical Components Theory

Example



Symmetrical Components Theory

Example (cont.)



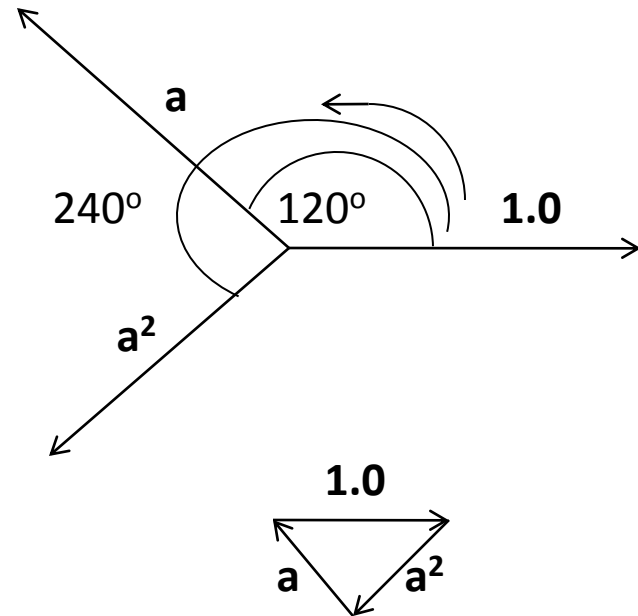
Symmetrical Components Theory

Unit Vector or Operator 'a'

Just as the operator j rotates the vector by 90° , we define a unit vector or operator 'a', which when multiplied to a vector rotates the vector through 120° in anticlockwise direction

$$\begin{aligned}a &= e^{j120} = \cos 120 + j \sin 120 \\&= -0.5 + j0.866 \\&= 1.0 \angle 120\end{aligned}$$

$$\begin{aligned}a^2 &= 1.0 \angle 120 \times 1.0 \angle 120 \\&= 1.0 \angle 240 \\&= -0.5 - j0.866 \\1 + a + a^2 &= 0\end{aligned}$$



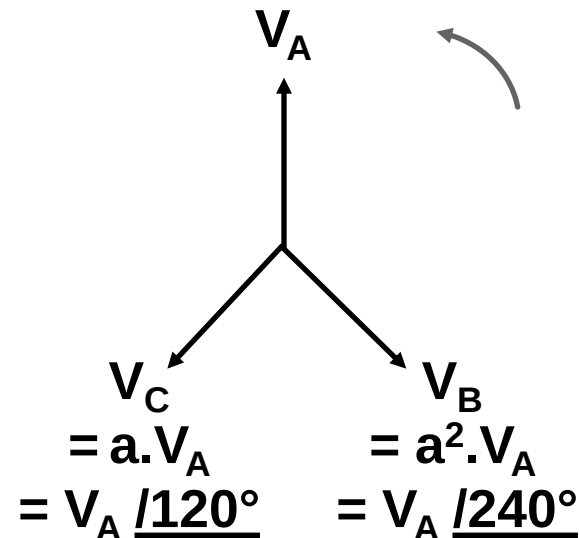
- If a vector is multiplied by operator 'a' the vector is rotated by 120°
- If a vector is multiplied by operator 'a^2' the vector is rotated by 240°

Symmetrical Components Theory

The “a” Operator

The “a” operator $(-0.5 + j\sqrt{3}/2)$ rotates a vector anticlockwise by 120°

e.g. balanced 3-phase system of positive sequence:



“a” operator is practical in deriving general equations for symmetrical components, by referring all quantities to a reference phase (V_A)

Symmetrical Components Theory

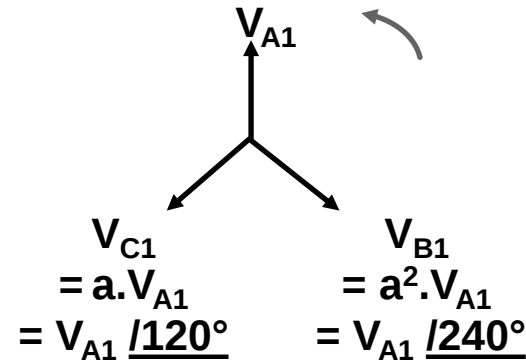
The “a” Operator

- Positive sequence system**

$$V_{A1} = V_{A1}$$

$$V_{C1} = a.V_{A1}$$

$$V_{B1} = a^2.V_{A1}$$

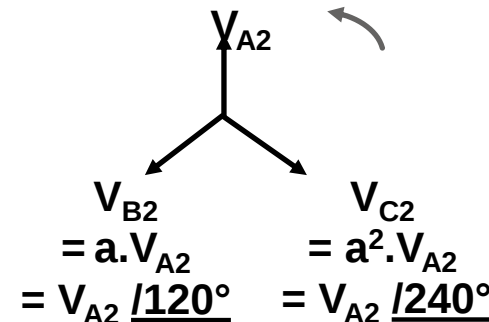


- Negative sequence system**

$$V_{A2} = V_{A2}$$

$$V_{B2} = a.V_{A2}$$

$$V_{C2} = a^2.V_{A2}$$

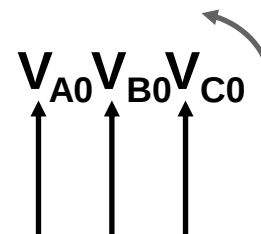


- Zero sequence system**

$$V_{A0} = V_{A0}$$

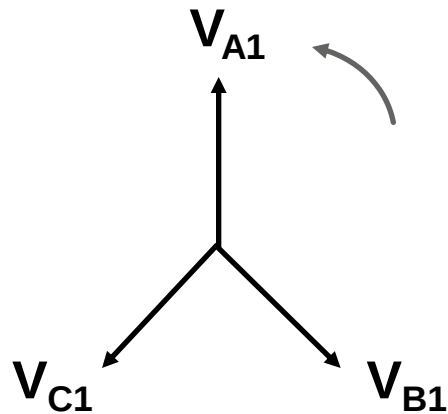
$$V_{B0} = V_{A0}$$

$$V_{C0} = V_{A0}$$

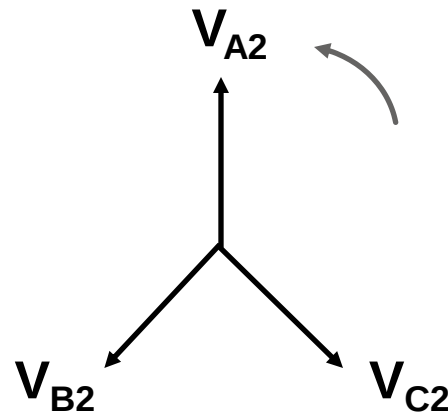


Symmetrical Components Theory

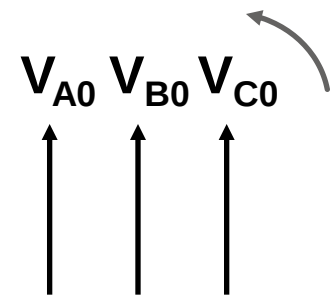
Phase Equations



**Positive Sequence
Component
(Normal Rotation)**



**Negative Sequence
Component
(Reverse Rotation)**



**Zero Sequence
Component
(In-phase)**

$$\begin{aligned} V_A &= V_{A1} + V_{A2} + V_{A0} \\ V_B &= V_{B1} + V_{B2} + V_{B0} = a^2.V_{A1} + a.V_{A2} + V_{A0} \\ V_C &= V_{C1} + V_{C2} + V_{C0} = a.V_{A1} + a^2.V_{A2} + V_{A0} \end{aligned}$$

Symmetrical Components Theory

Voltage Equations



$$V_{A1} = \frac{1}{3} \cdot (V_A + aV_B + a^2V_C)$$

$$V_{A2} = \frac{1}{3} \cdot (V_A + a^2V_B + aV_C)$$

$$V_{A0} = \frac{1}{3} \cdot (V_A + V_B + V_C)$$

$$V_A = V_{A1} + V_{A2} + V_{A0}$$

$$V_B = a^2V_{A1} + aV_{A2} + V_{A0}$$

$$V_C = aV_{A1} + a^2V_{A2} + V_{A0}$$

Symmetrical Components Theory

Current Equations

$$I_{A1} = \frac{1}{3} \cdot (I_A + aI_B + a^2I_C)$$

$$I_{A2} = \frac{1}{3} \cdot (I_A + a^2I_B + aI_C)$$

$$I_{A0} = \frac{1}{3} \cdot (I_A + I_B + I_C)$$

$$I_A = I_{A1} + I_{A2} + I_{A0}$$

$$I_B = a^2I_{A1} + aI_{A2} + I_{A0}$$

$$I_C = aI_{A1} + a^2I_{A2} + I_{A0}$$



Symmetrical Components

Practical Considerations

- Suppose the given voltages are balanced

$$V_A = V_A$$

$$V_B = \mathbf{a}^2.V_A$$

$$V_C = \mathbf{a}.V_A$$

Then

$$V_{A0} = \frac{1}{3}(1 + \mathbf{a}^2 + \mathbf{a})V_A = 0$$

$$V_{A1} = \frac{1}{3}(1 + \mathbf{a}^3 + \mathbf{a}^3)V_A = V_A$$

$$V_{A2} = \frac{1}{3}(1 + \mathbf{a}^4 + \mathbf{a}^2)V_A = 0$$

In a balanced system, zero and negative sequence components disappear, leaving only +ve sequence

Symmetrical Components

Practical Considerations

Under *BALANCED CONDITIONS* the system generates *ONLY POSITIVE SEQUENCE* currents and voltages

- normal operation
- three-phase faults

NEGATIVE SEQUENCE currents and voltages occur as a result of *UNBALANCE* on the system

- unbalanced loading
- unbalanced faults

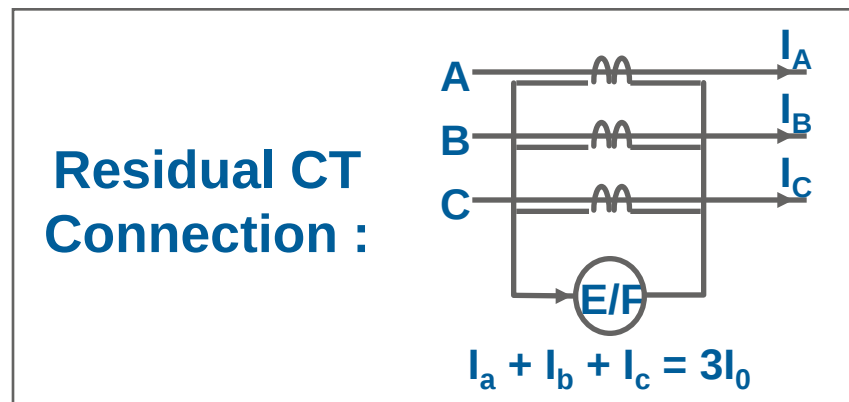
Symmetrical Components

Practical Considerations (cont.)

ZERO SEQUENCE currents and voltages occur as the result of an *EARTH FAULT* on the system

$$I_0 = \frac{1}{3} \cdot (I_A + I_B + I_C)$$

$$I_0 = \frac{1}{3} \cdot (\text{Residual Current})$$



Third harmonic components are also zero sequence quantities